FREE CONVECTIVE MASS TRANSFER AT DOWN-POINTING ISOSCELES TRIANGLES OF VARYING INCLINATION

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Free convective mass transfer at down-pointing triangular surfaces of different length and down-facing inclination was investigated using the limiting diffusion current technique of mass transfer measurement. The mass transfer rate was observed to be higher for a down-pointing triangular inclined surface over the entire range of inclination than that for an inclined rectangular surface because of the different "leading edge" condition. A mass transfer correlation for down-facing inclined triangular surfaces of different length was found to be $Sh_L = 0.774\,(Ra_L \cos\theta)^{0.25}$ for the $Ra_L \cos\theta$ range from $2 \cdot 10^6$ to $2 \cdot 10^{11}$.

Key words: Free convective mass transfer; Inclined isosceles triangles; Copper deposition.

Several previous accounts of work on free convective mass (heat) transfer at inclined surfaces[1–5] are known most of which made use of the limiting diffusion current technique (LDCT). Patrick *et al.*[1] studied free convection mass transfer to inclined down-facing rectangular surfaces. A plot of Sh_L against Ra_L gave separate straight lines for each inclination angle. However, inserting cos θ into the Grashof number, *Gr*, collapsed all the results for down-facing and vertical orientation onto a single straight line with very small scatter. For vertical and all inclined down-facing rectangular surfaces, the mass transfer results were well-correlated by the equation:

$$Sh_L = 0.68\,(Ra_L \cos\theta)^{0.25} \qquad (1)$$

for $6 \cdot 10^4 \le Ra_L \cos\theta \le 3 \cdot 10^9$. Flow visualization results were used to draw a flow regime map in which areas of different flow structures were indentified according to the inclination angle and Ra_L value. The exponent in Eq. (*1*), together with flow visualization results, indicates an attached, predominantly laminar, boundary layer flow at all inclinations.

The results of Fouad and Ahmed[2] for down-facing planes also follow Eq. (*1*) even up to the Ra_L cos θ value of 10^{12}, suggesting, rather surprisingly, that predominantly laminar flow persists up to this higher value of Rayleigh number.

Fuji and Imura[3] used two plates (30 cm height, 15 cm width and 5 cm height, 10 cm width) in the range of θ = –90° (down-facing) to θ = 90° (up-facing) for investigation of natural convection heat transfer. For the vertical (θ = 0°) and down-facing inclined plates, the average heat transfer data were correlated by the equation:

$$Nu_L = 0.56\,(Ra_L \cos\theta)^{0.25} \qquad (2)$$

for $10^5 \leq Ra_L \cos\theta \leq 10^{11}$.

Patrick and Wragg[4] studied the free convective mass transfer at cones with inactive base. The mass transfer results for the case of down-pointing cones, *i.e.*, with attached boundary layer, were correlated by the equation

$$Sh_L = 0.87\,(Ra_L \cos\theta)^{0.25}\ . \qquad (3)$$

Krysa and Wragg[5] used a limiting current technique for the measurement of the free convective mass transfer rate at entire triangular down-pointing pyramids and also at the individual faces of the pyramids. The cathodic deposition of Cu^{2+} ions at electrodes of varying geometries was used as the mass transfer process, the supporting electrolyte being sulfuric acid. The mass transfer rate at a single inclined triangular surface was correlated by the equation:

$$Sh_L = 0.78\,(Ra_L \cos\theta)^{0.25} \qquad (4)$$

for the Ra_L cos θ range from $9.3 \cdot 10^6$ to $2 \cdot 10^{11}$. The higher value of the coefficient in Eq. (*4*) for a single inclined triangular surface of a down-pointing pyramid, in comparison with Eq. (*1*) of Patrick *et al.*[1] for an inclined rectangular surface, was explained by the fact that the triangular surface is fed by a fresh electrolyte not only at the lower edge of the surface, as for a rectangular face, but all long the inclined edge of the triangle. This inevitably leads to a higher mean mass transfer coefficient.

The mass transfer correlation for all three inclined surfaces of the down-pointing pyramid, simultaneously active, gives the relationship

$$Sh_L = 0.714\,(Ra_L \cos\theta)^{0.25} \qquad (5)$$

for the same $R_L \cos \theta$ range as in Eq. (*4*). The coefficient in Eq. (*5*) is lower than that in Eq. (*4*) since the mass transfer rates at the single active surfaces are not additive because of interactions caused at the pyramid edges.

The geometric parameters of triangular inclined surfaces of down-pointing pyramids[5] (base *b*, length of inclined surface *L* and inclination angle θ) cannot be changed independently. Therefore a more detailed study of free convective mass transfer at single down-pointing isosceles triangular surfaces in different down-facing orientations was performed using the well-known LDCT of mass transfer measurement.

EXPERIMENTAL

The mass transfer experiments were performed in a Perspex tank with dimensions 20 × 20 × 24 cm. The cathodes were isosceles triangles. Table I lists all the geometric characteristics of the triangular surfaces used. This work considers a set of isosceles triangular surfaces with constant base (*b* = 2 cm) and varying length of inclined surface *L*. The possible mass transfer configurations for inclined triangular surface are shown schematically in Fig. 1a. Similarly to the work of Patrick *et al.*[1], the inclination angle for the down-facing surface was taken as negative. The cathodes were made of solid brass sheet of thickness 5 mm. Only one triangular side was exposed. The triangles were held in a position at a certain inclination angle by a 1.5 mm wire glued into the base of the triangle (Fig. 1b). This wire also provided an electrical connection for the cathode and was, together with the base and sides of triangle, insulated from the electrolyte with lacquer. In order to hold the wire and to provide different inclination angle settings, a support mechanism was designed and made up of Perspex. The wire ran through a hole in the arm of the supporting mechanism and was fixed by two Perspex screws. The arm could be fixed at different inclinations by a large screw. The counter electrode (anode) was a cylindrical copper mesh of diameter 18 cm and height 23 cm placed around the sides of a tank. The arrangement of the apparatus is similar to that described previously[6,7], and the supporting mechanism holding the triangle is schematically shown in Fig. 1b.

The Cu^{2+} concentration (c_b) varied from 30 to 238 mol m^{-3}. Each solution contained 1 500 mol m^{-3} sulfuric acid as a supporting electrolyte. The electrolyte temperature was carefully measured and al-

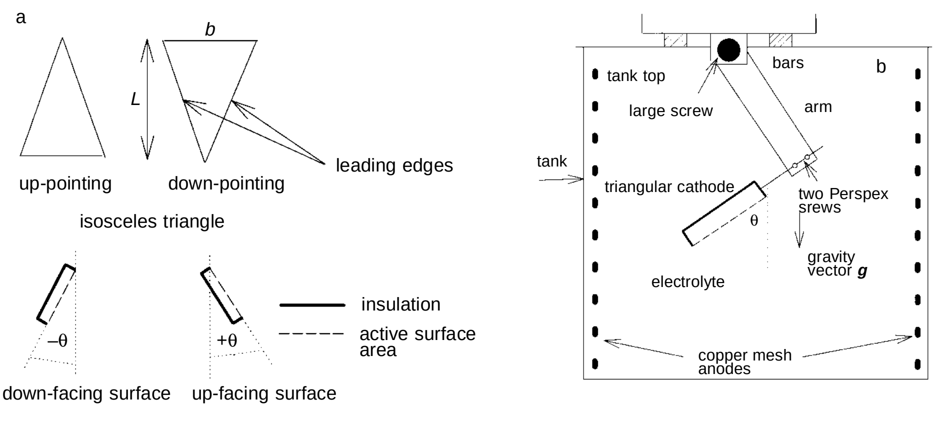

FIG. 1
Triangle configuration (a) and supporting mechanism for down-pointing isosceles triangles (b)

ways lay within the range of 18–21 °C being constant to ±0.1 °C during each individual experiment. The actual Cu^{2+} concentration was periodically determined by atomic absorption spectrophotometry (AAS).

The usual electric circuit for limiting current measurement was employed, consisting of a dc power supply with a voltage regulator, a high impedance voltmeter and a multi-range ammeter. Limiting currents were obtained by the well-known procedure, which has been reported in detail previously[8]. The anode acted as a reference electrode in view of its high area compared to that of the cathode. Under such conditions, polarization is negligible at the anode, and the cell current–voltage relationship depends only on the conditions prevailing at the cathode. The onset of the limiting current was sharp and reproducible.

RESULTS AND DISCUSSION

Mass Transfer Data Calculation

For each experiment, the mass transfer coefficient, k, was calculated from the measured limiting current, I_L, using the equation:

$$k = \frac{I_{\mathrm{L}}}{AnFc_{\mathrm{b}}} \ , \qquad (6)$$

where A is the area available for mass transfer and n is the number of electrons for Cu^{2+} ion reduction (n = 2). For the inclined triangular surfaces, the data were expressed in the form of a slant height Sherwood number and a slant height Rayleigh number

$$Sh_L = \frac{kL}{D} \qquad (7)$$

$$Ra_L = Gr_L\, Sc = \frac{\boldsymbol{g}\Delta\rho L^3 \rho}{\mu^2} \frac{\mu}{\rho D} = \frac{\boldsymbol{g}\Delta\rho L^3}{\mu D} \ . \qquad (8)$$

TABLE I
Geometric parameters of isosceles triangles b = 0.02 m (Fig. 1a)

Triangle	L/b	A, m^2
1	0.25	0.00005
2	0.49	0.000099
3	1.01	0.000202
4	2.02	0.000405
5	4.01	0.000803

This approach to data treatment was previously used for the correlation of free convective heat and mass transfer at inclined plates[1–3], cones with an insulated base[4] and pyramids[5,6].

The diffusivity of the Cu^{2+} ions was calculated using the data of Wilke *et al.*[9]. These data were used in preference to the equation of Fenech and Tobias[10] in view of the solution temperatures used. Electrolyte density and viscosity were calculated using the data of Eisenberg *et al.*[11]. The $\Delta\rho$ terms were taken from Wilke *el al.*[9]. The effect of migration on the copper deposition rate was negligible; the migration contribution for the highest concentration of copper sulfate (250 mol m^{-3}) was 1.5% (ref.[12]).

Mass Transfer Measurement

The effect of inclination angle on the mass transfer coefficient for a down-pointing triangular surface of constant base and different length is shown in Fig. 2 for Cu^{2+} concentration 120 mol m^{-3}. It is apparent that the mass transfer coefficient decreases with the triangle length as well as with its inclination from the vertical to the downward horizontal orientation.

The effect of length of the inclined surface on the mass transfer coefficient for a down-pointing triangular isosceles surface is shown in Fig. 3 for different inclination angles and for two Cu^{2+} concentrations 30 and 250 mol m^{-3}. In all the cases, the mass transfer coefficient decreases with increasing length and with decreasing inclination angle. When the triangle is inclined at –79°, there is a strong decrease in the mass transfer coefficient rate with the length, *L*, in the range of 0.5–1(2) cm and then the mass transfer coefficient decreases very slowly.

The mass transfer coefficient at isosceles triangles inclined at –45° decreases with length similarly to vertical triangles. The difference between the mass transfer coefficient for triangles in vertical and inclined (–45°) orientations is very small. This fact

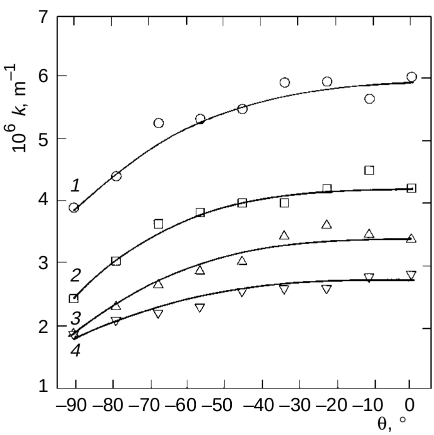

FIG. 2
Effect of inclination angle, θ, on the mass transfer coefficient, *k*, for a down-pointing isosceles triangular surface of constant base and different length; Cu^{2+} ion concentration 120 mol m^{-3}. Length of inclined surface L = 0.005 m (*1*), 0.0099 m (*2*), 0.0405 m (*3*), 0.0803 m (*4*)

suggests that the boundary conditions at inclination –45° are similar to those for vertical orientation and the hydrodynamic regime of the laminar boundary layer does not change to turbulent flow.

A comparison of the effect of length on the mass transfer coefficient at vertical rectangular, vertical down-pointing isosceles triangular and vertical cylindrical surfaces is shown in Fig. 4. It can be seen that the mass transfer coefficient is slightly higher in case of isosceles triangular surfaces than that for rectangular or cylindrical surfaces. The situation can be explained by the fact that the "leading edges" (Fig. 1a) of down-pointing isosceles triangular surfaces are different from those for rectangular surfaces and the triangular surface is exposed to fresh solution not only at the horizontal bottom

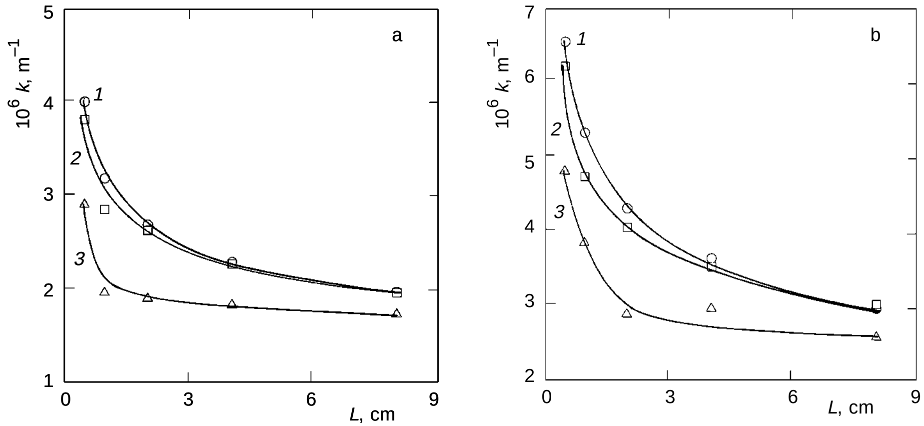

FIG. 3
Effect of length of the inclined surface, *L*, on the mass transfer coefficient, *k*, for a down-pointing isosceles triangular surface of different inclination angle from vertical; Cu^{2+} ion concentration 30 mol m^{-3} (a) and 250 mol m^{-3} (b). Inclination angle θ = 0° (*1*), –45° (*2*), –79° (*3*)

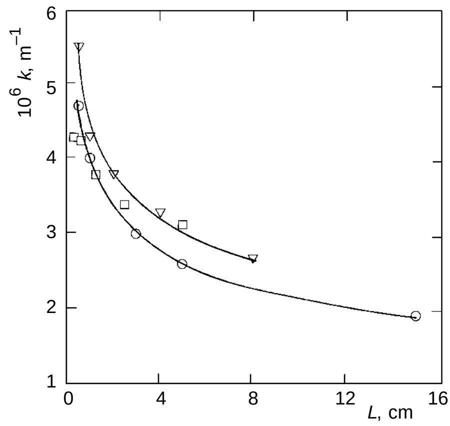

FIG. 4
Effect of length of the inclined surface, *L*, on the mass transfer coefficient, *k*, at vertical rectangular (❒), vertical down-pointing isosceles triangular (∇) and vertical cylindrical (❍) surfaces; Cu^{2+} ion concentration: 106 mol m^{-3} (❒), 120 mol m^{-3} (∇), 129 mol m^{-3} (❍)

edge as with a rectangular or cylindrical surface but also at the inclined edges along the length of the triangular surface.

The decrease in mass transfer coefficient with increasing down-facing inclination was reported by Patrick *et al.*[1] for rectangular plates. Therefore the mass transfer coefficients for isosceles triangular inclined surfaces were compared with those for rectangular surfaces. The effect of length of the inclined surface on the mass transfer rate at inclined down-facing (–45 and –79°) rectangular and isosceles triangular surfaces is shown in Fig. 5. The mass transfer rate is distinctly higher for triangular surfaces than that for rectangular surfaces. The reason is again the favourable leading edge condition for triangular surfaces. Figures 4 and 5 show that the difference in the mass transfer

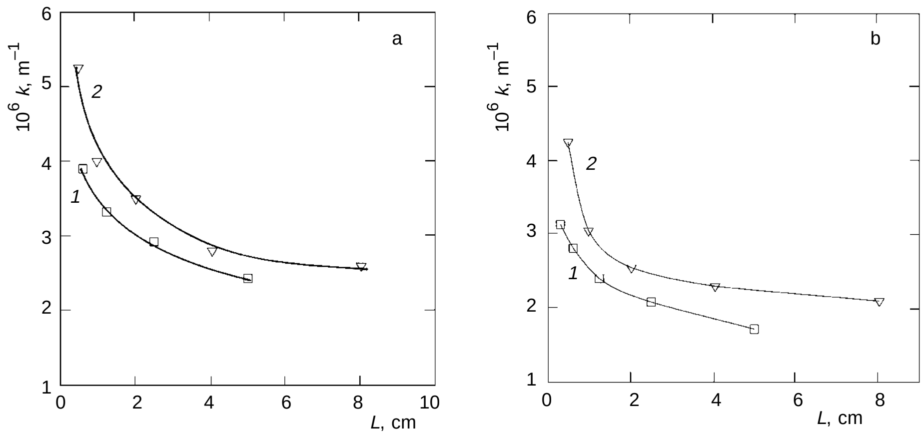

FIG. 5
Effect of length of the inclined surface, L, on the mass transfer coefficient, k, at inclined down-facing –45° (a) and –79° (b) rectangular (*1*), and isosceles triangular (*2*) surfaces; Cu^{2+} ion concentration 120 mol m^{-3}

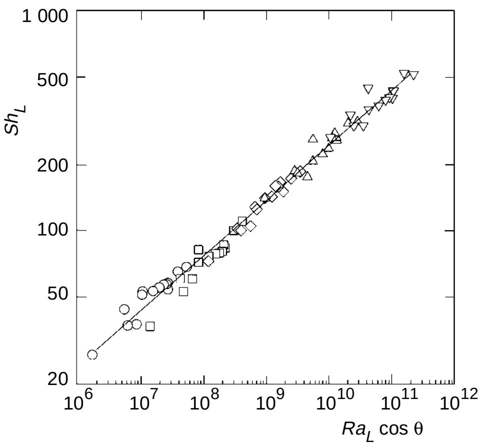

FIG. 6
Overall mass transfer correlation for inclined down-facing isosceles triangles. L = 0.5 cm (❍), 0.99 cm (❐), 2.02 cm (✧), 4.05 cm (Δ), 8.03 cm (∇). Full line from Eq. (*10*)

rate between triangular and rectangular surfaces increases with increasing down-facing orientation from the vertical. This is likely due to the fact that rectangular surfaces studied by Patrick *et al.*[1] were placed in a coplanar Perspex plate so that the surface was in fact surrounded by a coplanar isolated surface. As shown in recent work[13], the mass transfer rate to down-facing collared horizontal discs is significantly lower than that for uncollared discs. It can be assumed that the effect of the collar on the mass transfer rate for the rectangular surface is therefore significant when inclination is close to the down-facing horizontal while at inclination –45° and for the vertical orientation, the effect of the collar is very small.

An overall mass transfer correlation for down-pointing inclined triangular surfaces of different length, L, is shown in Fig. 6 using $Ra_L \cos \theta$ and Sh_L numbers defined by Eqs (*8*) and (*7*), respectively. A least square analysis of 70 points gives

$$Sh_L = 0.756\,(Ra_L \cos \theta)^{0.251} \quad . \tag{9}$$

Forcing a slope 0.25 gives the relation

$$Sh_L = 0.774\,(Ra_L \cos \theta)^{0.25} \tag{10}$$

for the $Ra_L \cos \theta$ range $2 \,.\, 10^6$ to $2 \,.\, 10^{11}$. The coefficient 0.774 was reached with the 95% confidence limits of 0.752–0.796. The coefficient of correlation $r^2 = 0.982$.

The coefficient in Eq. (*10*) is higher than that in Eq. (*1*) for rectangular plates. This result is consistent with the correlating equation (*4*) for single inclined triangular surfaces of down-pointing pyramids. Equation (*10*) confirms that the mass transfer rate is higher for a down-pointing triangular inclined surface (in the whole range of inclination) than for a rectangular surface because of the different "leading edge" conditions. The correlating equation (*10*) can be used to predict free convective mass transfer at inclined down-pointing isosceles triangular surfaces of varying base b and L/b ratio in the range of 0.25–4.0.

SYMBOLS

A	surface area of triangular electrode, m^2
b	base of triangular surface, cm
c_b	bulk concentration of Cu^{2+} ions, mol m^{-3}
D	diffusion coefficient of Cu^{2+} ions, $m^2\,s^{-1}$
F	Faraday constant, 96 487 C mol^{-1}
$\boldsymbol{g}$	gravitational acceleration, m s^{-2}
Gr_L	Grashof number based on inclined surface length, $Gr_L = \Delta\rho \boldsymbol{g} L^3 \rho/\mu^2$
h	coefficient of heat transfer, W $m^{-2}\,K^{-1}$

I_L	limiting diffusion current, A
k	mass transfer coefficient, m s^{-1}
K	thermal conductivity, W m^{-1} K^{-1}
L	length of inclined surface, m
n	charge number of Cu^{2+} ion (n = 2)
Nu_L	Nusselt number based on inclined surface length, $Nu_L = hL/K$
Ra_L	Rayleigh number based on inclined surface length, Eq. (*8*)
Sc	Schmidt number, $Sc = \mu/\rho D$
Sh_L	Sherwood number based on inclined surface length, Eq. (*7*)
T	electrolyte temperature, K
$\Delta\rho$	density difference between bulk solution and interface, kg m^{-3}
ρ	density, kg m^{-3}
μ	dynamic viscosity, kg m^{-1} s^{-1}
θ	inclination angle from the vertical, °

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